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Abstract

The graceful degradation properties of matched isolated power combiners as a function of amplifier failures of the same type has been determined using general techniques applicable to all such circuits. The effects of complete amplifier failures, amplitude and phase imbalances and poor VSWRs on overall combiner system output power, combining efficiency and input-output impedances can be determined by the resulting equations and plots.

Introduction

Pairs of microwave amplifiers are frequently combined through quadrature couplers to achieve low VSWRs at the input and output interfaces.^{1,2} Furthermore, many microwave applications require the use of power combining networks to achieve solid state amplifier power levels many times greater than is currently available from single devices. In both these cases, it is vital to know the effects of amplifier gain imbalances, phase imbalances and failures upon the overall power system, but this data has not been previously published. A general analysis will be presented here which gives a quantitative prediction of the influence of arbitrary deviations from the ideal gain and phase shift of an arbitrary number of amplifiers in a divider, N-amplifier, combiner system. The effects on overall power output, combining efficiency, and input and output impedance levels will be shown.

As individual amplifiers fail in an N-amplifier system, as shown in Figure 1, it is highly desirable to have graceful degradation. This means that the operational amplifiers are not affected by the failures, and that the greatest possible power be delivered to the output port. In certain applications, the graceful degradation properties make N-amplifier systems attractive over a single amplifier when failures are considered. In the ideal case, all divider output ports and combiner input ports are isolated from each other, and all ports are matched. These networks may be either reciprocal or non-reciprocal.

Power Output Degradation With Total Amplifier Failures

Consider such an N-amplifier system in which m amplifiers fail leaving M = N-m amplifiers operational. Before failure, under ideal conditions, the total E-field at the output port of a combiner is simply the scalar sum of the N equal level in-phase components resulting from each of the N amplifiers. When only M amplifiers are operational, the output E-field decreases to M/N of its original value. Ideally, it would be hoped that output power would also be directly proportional to the number of working amplifiers, or, equivalently, that the total output power of a combiner would be equal to the input power from the M operating amplifiers. However, since the output power is proportional to the square of the E-field, it falls as the square of the number of working amplifiers.

Therefore, the power output relative to the maximum output power is given by (M/N)². This means that in the case where half the amplifiers fail, the input power drops to half of its original level, but the output goes to one-quarter of its original level. Thus, if one amplifier fails in a balanced amplifier, the output drops by six dB rather than three dB.

The combining efficiency as a function of amplifier failures can be defined as the ratio of combiner output power to the total combiner input power. This network efficiency parameter gives a measure of how much of the amplifier power available at all the input ports reaches the combiner output. Realizing that in the ideal case, the maximum input power is equal to the maximum output power when all stages are operative it can be seen that since the relative input power is given by M/N and the relative output power is given by (M/N)², that the combining efficiency, η_c , is given by M/N. Thus, when half the amplifiers fail, the combining efficiency is 50%.

Partial Amplifier Output Failures

When all the amplifier outputs in Figure 1 are properly phased, the combiner output is given by^{3,4}

$$P_o = \frac{1}{N} (\sqrt{P_1} + \sqrt{P_2} + \dots + \sqrt{P_n})^2 \quad (1)$$

Under normal operating conditions, consider the output power of an individual amplifier of the system to be P_{norm} so that the combiner output becomes NP_{norm} . If an amplifier fails, its output will be expressed as kP_{norm} where k represents the fractional output of a failing amplifier and varies between 0 and 1.

Consider the case where m amplifiers fail to an output of kP_{norm} leaving M = N-m amplifiers operating at full output. The output power is then given by

$$P_o = \frac{1}{N} (M \sqrt{P_{norm}} + m \sqrt{kP_{norm}})^2 = \frac{P_{norm}}{N} (M + m \sqrt{k})^2 \quad (2)$$

and the total combiner input power is given by

$$P_{in} = MP_{norm} + mkP_{norm} = P_{norm} (M + mk) \quad (3)$$

Realizing that the maximum output power is given by $P_{om} = NP_{norm}$, the relative output power of an N-way combiner is given by

$$\frac{P_o}{P_{om}} = (\sqrt{k} + \frac{M}{N} (1 - \sqrt{k}))^2 \quad (4)$$

which is plotted in Figure 2. Combining efficiency, η_c , is defined as the ratio of the total output power of a combiner to the total input power. By taking the ratio of equation 2 to equation 1, the combining efficiency is found to be given by

$$\eta_c = \frac{(\frac{M}{N} (1 - \sqrt{k}) + \sqrt{k})^2}{\frac{M}{N} (1 - k) + k} \quad (5)$$

which is plotted in Figure 3.

To illustrate the usefulness of these general relations, consider the example of a four-way combiner using 15 dB gain amplifier modules. Two cases will be considered. In the first case, three amplifiers will fail completely so that $k = 0$. In the second case, three amplifier modules will be replaced by unity gain connections so that $k = 0.0316$, which corresponds to a gain reduction of 15 dB. Applying equations 4 to 6 results in the following tabulation

Parameter	Case I	Case II
k	0	0.0316
$\frac{M}{N}$	0.25	0.25
$\frac{P_o}{P_{om}}$	0.0625	0.144
η_c	0.25	0.5369
P_{in}	P_{norm}	$1.09 P_{norm}$
P_o	$0.25 P_{norm}$	$0.59 P_{norm}$

This example shows that in this case if the failed amplifiers are replaced with properly phase unity gain modules, more than twice the output power will be obtained.

The data for the two-way combiner case can be very valuable in the design of balanced amplifier stages. The combining loss can be readily determined for the case of two amplifiers having unequal output power levels by using equation 5 and Figure 2. Thus, if there is a 1 dB difference between amplifiers, $k = 0.972$, and $\eta_c = 0.997$. For a three dB difference, $k = 0.501$ and $\eta_c = 0.972$. Thus, it is seen that only a small penalty is paid for making a balanced amplifier with two amplifiers which are not very closely matched in output power level.

Phase Tracking Considerations

In this section, failing amplifiers will be considered as maintaining full output power, but having a phase shift of \emptyset degrees away from the correct value. When m amplifiers fail, the E-field at the output of the combiner will consist of two out of phase components. The output power is found to be given by⁴

$$P_o = \frac{P_{norm}}{N} (M^2 + 2 M m \cos \emptyset + m^2). \quad (6)$$

Realizing that in this case $P_{in} = P_{om} = N P_{norm}$, the combining efficiency is identical to the relative output power, or

$$\eta_c = \frac{P_o}{P_{om}} \quad (7)$$

Hence, only a single expression and a single set of graphs is needed for this case. The expression for the combining efficiency can be obtained by dividing equation 6 by $N P_{norm}$. After some algebraic manipulation, the combining efficiency is found to be

$$\eta_c = 1 - 2 \frac{M}{N} (1 - \frac{M}{N}) (1 - \cos \emptyset) \quad (8)$$

which is shown in Figure 4.

For the balanced amplifier case this becomes

$$\eta_c = \frac{1 + \cos \emptyset}{2} \quad (9)$$

In this case, this data shows that two combined amplifiers can be as much as 45 degrees out of phase with only 1 dB combining loss. If there is a phase difference as great as 90 degrees, the combining loss goes up to 3 dB. The effects of phasing errors are greatest for the binary case ($M/N = 0.5$) as can be seen in Figure 4.

Input-Output Impedances With Amplifier Failures

The effects of amplifier failures on the input and output impedances of an amplifier combiner system can be determined by using a scattering parameter analysis of the system shown in Figure 1. The input divider will be analyzed realizing that the same techniques are applicable to the output combiner. If it is initially assumed that all amplifiers have an input reflection coefficient of ρ_k , then the divider input reflection coefficient is given by

$$\begin{aligned} \frac{b_o}{a_o} &= \frac{\rho_k}{N} (e^{j2(\theta_1 + \dots + 2\theta_n)}) \\ &= \frac{\rho_k}{N} e^{-j2\theta_1} (1 + e^{j2(\theta_2 - \theta_1)} + \dots + e^{j2(\theta_n - \theta_1)}) \end{aligned} \quad (10)$$

Hence, it can be seen that the input reflection coefficient is a function of the relative phasing between outputs of the divider $(\theta_1 - \theta_1)$.

If the relative phasing between outputs is either in phase $(\theta_1 - \theta_1 = 0^\circ)$ or antiphase $(\theta_1 - \theta_1 = 180^\circ)$, all of the exponential terms in equation 10 become unity, and the total reflection coefficient becomes

$$\frac{b_o}{a_o} = \rho_k \quad (11)$$

If M amplifiers are considered operational with $\rho_k = 0$, and the remaining $N-M$ amplifiers fail with $\rho_k = 1$, the reflection coefficient at the composite input becomes

$$\frac{b_o}{a_o} = \frac{N-M}{N} = 1 - \frac{M}{N}$$

This worst case condition is shown in Figure 5(a).

If the divider outputs are phased in multiples of 90° , then the exponent terms can be either +1 or -1.

Assume, as before, the worst case condition that a working amplifier has $\rho_k = 1$. From equation 10, it can be seen that when amplifiers fail in pairs that are driven in phase quadrature with respect of each other

$$\frac{b_o}{a_o} = 0.$$

If amplifiers fail in pairs driven either in-phase or in anti-phase, then

$$\frac{b_o}{a_o} = 1 - \frac{M}{N}$$

as before. However, when greater than half the amplifiers fail, there must be some amplifier pairs which are driven in phase quadrature. In this case, the maximum reflection coefficient is equal to M/N . Therefore, the maximum reflection coefficient for the case of fewest possible cancelling reflections is given by

$$\frac{b_o}{a_o} = \frac{M}{N}, \quad (0 \leq \frac{M}{N} \leq 0.5) \quad (12a)$$

$$\frac{b_o}{a_o} = 1 - \frac{M}{N}, \quad (0.5 \leq \frac{M}{N} \leq 1.5) \quad (12b)$$

This relation is given in Figure 5(b). It can be seen that the reflection coefficient has a maximum value of 0.5 when $M/N = 0.5$.

The preceding analysis shows that the use of quadrature dividers and combiners within a larger divider/combiner network minimizes reflection coefficients when amplifier stages begin to fail. This in turn reduces the mismatches presented to the driving amplifier stages and at the output of the high level amplifier, which is frequently an impedance sensitive element such as an antenna.

Summary

The power output, efficiency, and input-output VSWRs of microwave matched, isolated N-way power ampli-

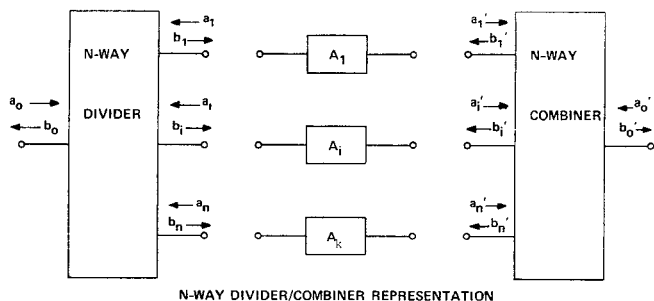


Figure 1. Divider, N-Amplifier, Combiner System.

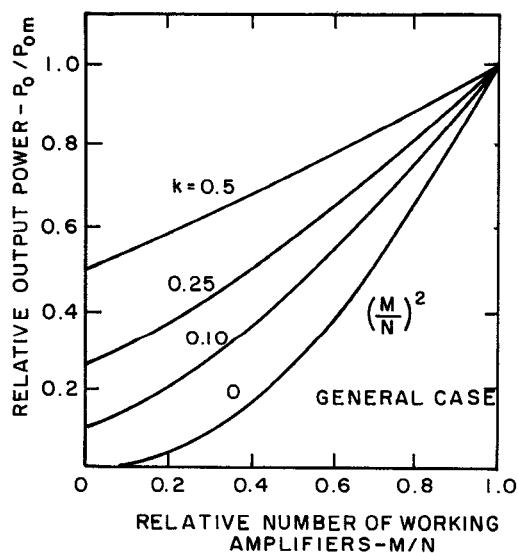


Figure 2. Relative Power Output of an N-way Combiner as a Function of Amplitude Errors.

fier combiners are determined as a function of identical amplifier amplitude or phase imbalances. In the case of complete amplifier failures, this type of combiner follows a predictable $(M/N)^2$ graceful degradation law.

Quadrature couplers are desirable in power divider/divider/combiners to minimize overall input/output mismatches. When pairs of amplifiers fail that are driven in phase quadrature, the overall system remains matched.

References

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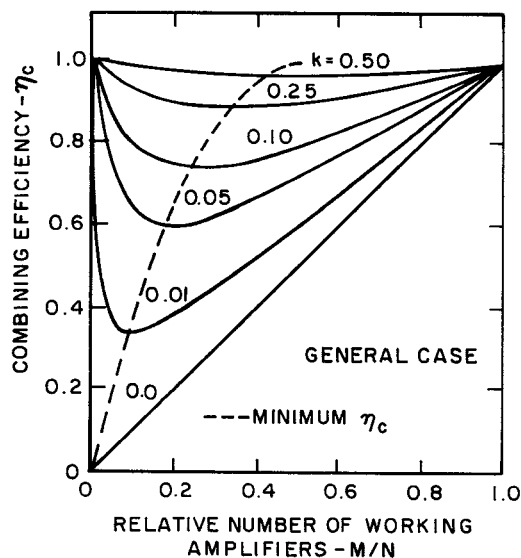


Figure 3. Combining Efficiency of an N-way Combiner as a Function of Amplitude Errors.

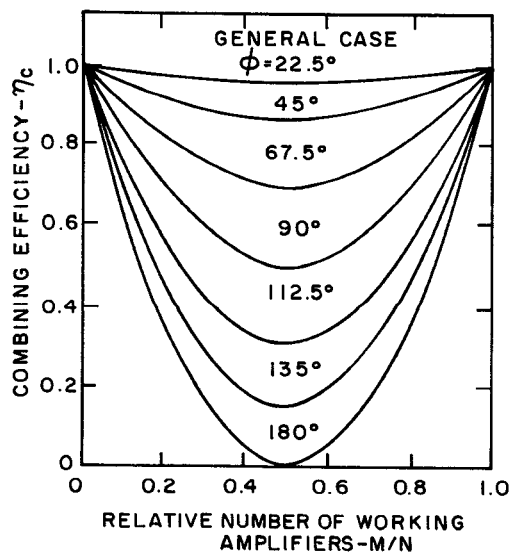
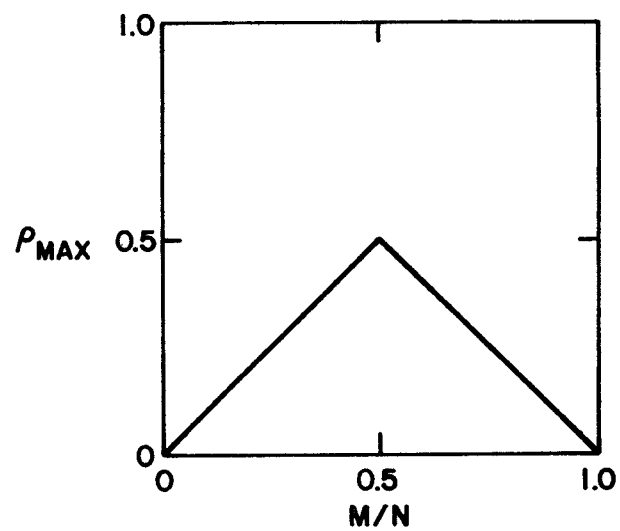
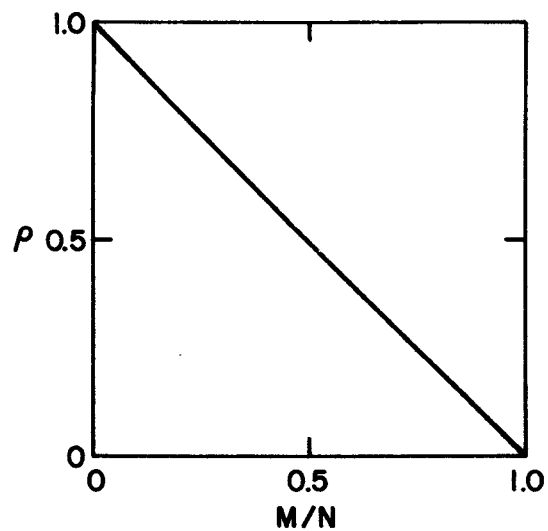


Figure 4. Combining Efficiency of an N-way Combiner as a Function of Phasing Errors.



(a) In-phase (0°) and Anti-phase (180°) Cases.

(b) Quadrature Cases.

Figure 5. Reflection Coefficient (ρ) as a Function of Relative Number of Working Stages (M/N).